Unions and market integration in contests*

Frode Meland†

Abstract

In this paper we study the effects of introducing endogenous costs in a Tullock model of rent-seeking. We show that unions can be efficiency improving, and that the firms' levels of effort depend more critically upon the number of firms participating in the contest when unions are present. We then study the effects of market integration in a two-country setup. Integrating two initially separate markets is shown to decrease union set wages, but is nevertheless beneficial to firms of both countries only if there are sufficiently few contestants. However, unions and firms in one country might benefit from integration if their resident country is sufficiently large compared to the post-integration market.

Keywords: contest, union, market integration

JEL classification: F12, F15, J51, L13

1 Introduction

This paper uses contest theory to study how unionization and market integration affect firms submitting tenders for projects. A Tullock contest (Tullock (1980)) is used as the basis of our analysis.[1] Firms are thus assumed to exert some level of costly effort to win a prize, to be interpreted in this setting as a project awarded. The analysis is twofold. We start out by investigating

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†University of Bergen, Fosswinckelsgt. 6, N-5007 Bergen, Norway. E-mail: frode.meland@econ.uib.no

[1] For a detailed introduction to the literature of rent-seeking, see for instance Nitzan (1994).

the implication of unionization in a single contest market.[2] This is followed by an analysis of integration of two such markets. We find that the presence of unions will typically reduce the value of the projects to the firms as labor costs are pushed up. This directly reduces the effort a firm is willing to exert, and can thus constitute a welfare improvement if effort is wasteful. In a monopoly union model with firm level bargaining, it is shown that total effort is more dependent on the number of firms in the presence of unions, compared to the non-union case. This constitutes a potentially important result in its own right, as it could be of significance for a contest manager looking to keep effort at a desired level.

Turning to contest market integration, we study the integration of two such unionized economies, not necessarily of equal sizes. In our set-up, there is one project awarded in each country, and all firms can participate in the contests for both prizes when markets are integrated.[3] The integration process leads unions unambiguously to choose lower wages, but given sufficiently large differences in country sizes, measured by the number of firm/union pairs, unions of the large country could actually benefit from such an integration. Unions in the larger country already had the lowest wages before integration, and integration doubles the available projects at the expense of less than double the competition. Because of the effect on wages, it is possible for firms of both countries to benefit from integration, but this only happens if there are very few contestants (2 in each country). If markets differ in size, firms of the larger country can benefit from integration for basically the same reasons as the unions.

The following section defines a Tullock contest for a single prize in a unionized environment, and finds the equilibrium effort levels of firms given wages by the firm-specific unions. This part of the analysis is similar to Hillman and Riley (1989). We then go on to specify union behavior in section 3,

[2] Union influence on contests has also been studied by Rama (1997) and Straume (2001). These papers consider the effects of unionization on efforts to uphold or achieve monopoly rights by firms. The set-up in our model is quite different, but the initial analysis has one similarity: The existence of unions is shown to decrease the effort exerted by firms.

[3] In a contest setting, the problem of market integration takes on a special form. Usually, either a contest is open for firms of another country or it is not. Trade costs are likely to be of less importance. Also, in this era of free trade organizations, there may be little scope for domestic authorities to exclude foreign firms from participating in bidding for domestic projects. For example, all countries within the European Economic Area are required to keep the bidding rounds for substantial projects open to all firms in the EEA. This can be viewed as an integration of contest markets, some implications of which we investigate in this paper. Konrad (1999) also studies contests in an international context, though the focus there is on trade and optimal trade policies.

finding the implications of unionized labor on the Tullock game equilibrium. In section 4 we expand the model to the case of two projects being on offer in an integrated market and discuss the effects of market integration. Section 5 presents our conclusions.

2 The Tullock single prize contest

Utilizing the standard Tullock contest of rent-seeking as our basis, we define a prior stage where some firm-specific costs are endogenously determined. We adopt a model in which unions choose a wage level that directly affects each firm's value of the good that they bid for. It follows that the wages, both in absolute and relative terms, determine the effort of each firm and thus the equilibrium expected outcomes.

We assume that gross of labor costs, the firms' evaluations of the project all equal V. Furthermore, the project necessitates the use of a fixed amount of labor. This simplifying assumption leads each union to be concerned with wages only, and the total labor costs for firm i can be normalized to the wage level, w_i, which is assumed binding for the firm and union. Thus, the net valuation of firm i is given by $V - w_i$. There are $n \geq 2$ firms.

We assume a Tullock success probability of the form

$$P(x_i, S_n) = \frac{x_i}{S_n}, \tag{1}$$

where x_i denotes the effort by firm i and $S_n = \sum_{j=1}^{n} x_j$. The effort levels are all sunk at the time that the project is awarded. The efforts might be thought of as costs sunk into development of a product that the contest manager may or may not choose to order.[4] The subsequent production then yields profits of $V - w_i$. This structure is essentially that of a research tournament and might not be relevant for situations where firms just stipulate a price for a given project and the winner is the lowest bidder. However, for many projects, especially public projects of some size, our structure encompasses another important aspect: When the authorities announce that they, say, intend to build an underwater tunnel, it would typically be up to the potential contractors to define the project in terms of construction techniques, surrounding infrastructure and even location. The project is then *defined* by each firm, and the authorities would have to compare the bids on the basis of numerous factors in addition to cost. For complex projects, there might not be any way for the involved parties to know what project will be preferred by the authorities. Nevertheless, it is reasonable to assume that putting more

[4] We could also think of the effort as lobbying effort with an aim to obtain a project.

effort into the development of the project will increase a firm's chance of being successful. We believe this kind of structure to be relevant and best described by a non-discriminatory contest as the one presented here.

We assume that the firms all choose effort levels simultaneously under the Cournot-Nash conjecture.[5] The analysis of the firms' maximization problems is then well known from the literature, and we therefore keep this section to a minimum.

When choosing effort, each firm maximizes expected profits:

$$x_i = \arg\max_{x_i} P(x_i, S_n)(V - w_i) - x_i. \tag{2}$$

The first order condition is simply[6]

$$x_i = S_n - \frac{S_n^2}{V - w_i}. \tag{3}$$

Aggregating over all firms, we obtain

$$S_n = \frac{n-1}{\sum_{j=1}^{n} \frac{1}{V - w_j}}. \tag{4}$$

For later reference, total rent seeking in the case of zero cost of labor to all firms, is

$$S_n = \frac{n-1}{n} V. \tag{5}$$

We assume that there is a fixed number of firms participating in the contest. This assumption is necessary given our way of modelling union behavior, as will be elaborated on in the following section.

3 Wage setting in a single prize market

Firm specific monopoly unions are assumed to set wages simultaneously to maximize expected utility,[7]

$$E[U_i] = w_i P(x_i, S_n). \tag{6}$$

[5] We could also have modelled efforts to be chosen sequentially. However, we have chosen the simultaneous approach because, in the case of large projects, the planning and development stage might extend over a long period of time – making a sequential setup implausible.

[6] It is easily shown that the second order condition for profit maximization is always satisfied.

[7] Again, we could have modelled a sequential set-up. However, as we will later model integration of two contest markets and it is not at all clear how the sequence of wage setting should then be modelled, we choose the simultaneous approach.

Here, w_i is the excess wage, where the alternative wage level is normalized to zero. If the relevant firm is not awarded the project, union utility is assumed to be zero. This could be thought of as the alternative use of the labor force being production of some good at the competitive wage level (here: $w_i = 0$).[8]

With the present set-up, any union would find it profitable to reduce wages by so much that its firm actually enters the contest (given low entry costs). Thus we assume that there is some exogenous restriction on the number of firms.[9]

The first order condition for each union is

$$\frac{\partial E[U_i]}{\partial w_i} = P + w_i \frac{\partial}{\partial w_i} P = 0. \tag{7}$$

In the appendix it is shown that

$$\frac{\partial}{\partial w_i} P = \frac{S_n}{(V - w_i)^2} [\frac{S_n}{(n-1)(V - w_i)} - 1]. \tag{8}$$

Inserting this expression into the optimum condition, imposing a symmetric equilibrium $w_i = w$, and consequently $x_i = \frac{1}{n} S_n$, where $S_n = \frac{n-1}{n}(V - w)$, we arrive at:[10]

$$\frac{\partial E[U_i]}{\partial w_i} = \frac{1}{n} - w \frac{(n-1)^2}{n^2 (V - w)}. \tag{9}$$

Solving for $\frac{\partial E[U_i]}{\partial w_i} = 0$, we get

$$w = n \frac{V}{n^2 - n + 1}. \tag{10}$$

We can now state our first result:

Proposition 1 *The existence of unions leads to lower effort in equilibrium.*

Proof. In equilibrium, $S_n = \frac{n-1}{n}(V - w)$, which is lower than for the non-union case given by (5). ■

[8] Using $U = w_i^a P(x_i, S_n)$, where $a > 0$, would not qualitatively change the results in this section.

[9] The set-up assumes unions to be able to commit to a wage level. If they are not, union influence will simply add to the equation through a constant diminished valuation for each project by the firms. However, through the reputation it creates, unions might find it profitable to act non-opportunistically to induce the firm to bid aggressively in future contests.

[10] This is the unique symmetric equilibrium, as shown in a supplementary appendix which can be downloaded from www.econ.uib.no/stab/frode/pc.pdf.

This result is hardly surprising, and it stems directly from the fact that the existence of unions reduces the value of the prize to the firms. However, it also implies that the existence of unions could lead to a higher level of market efficiency if effort is wasteful.

The next result might seem equally intuitive, but the explanation behind it is not as transparent:

Proposition 2 *The unions' wage level and expected utility decrease with the number of firm/union pairs.*

Proof. Differentiating (10) with respect to n yields

$$\frac{\partial w}{\partial n} = V\frac{n^2 - n + 1 - n(2n-1)}{(n^2 - n + 1)^2} = V\frac{1 - n^2}{(n^2 - n + 1)^2} < 0. \tag{11}$$

Furthermore,

$$\frac{\partial E[U]}{\partial n} = \frac{\partial}{\partial n}(\frac{1}{n}w) = \frac{1}{n}\frac{\partial w}{\partial n} - \frac{w}{n^2} < 0. \tag{12}$$

■

Ceteris paribus, the firms are faced with more competitors as the number of firm/union pairs is increased. This leads to a lower probability of any single firm being awarded the project. However, this level effect on the probability of success is *not* the reason why unions choose to lower wages. They do so because the probability of success is becoming more responsive to changes in their own wages.[11] Thus every union has an incentive to reduce wage demands, and the equilibrium entails both lower wages and lower probability of success due to the increase in the number of competitors. Consequently union utility is reduced.

Finally we have:

Proposition 3 *Rent seeking is more responsive to the number of firms when unions are present.*

Proof. Without unions, we had

$$\frac{\partial}{\partial n} S_n|_{\text{no union}} = \frac{V}{n^2}. \tag{13}$$

[11] This is illustrated in the appendix.

Now,

$$\frac{\partial}{\partial n}S_n|_{\text{union}} = \frac{\partial}{\partial n}V\frac{n-1}{n}[\frac{(n-1)^2}{n+(n-1)^2}]$$
$$= \frac{V}{n^2}\frac{(n-1)^2(3n[n+(n-1)^2]-[2n+(n-1)(3n-1)][n-1])}{[n+(n-1)^2]^2}$$
$$= \frac{V}{n^2}\frac{(n-1)^2(n(n+2)+(n-1)^2)}{[n+(n-1)^2]^2}. \tag{14}$$

Straightforward algebra shows the second fraction to be larger than 1. ■

The intuition behind this result is simple: As we have seen, an increase in the number of firms reduces equilibrium wage demands. This induces a greater increase in effort as the number of firm/union pairs increase than would be the case if wages were exogenous.

Following the above result, a deviation from the optimum number of firms would have a larger effect on effort compared to the non-union case. If high levels of effort is the objective, the inclusion of another contesting firm will increase effort more in a unionized setting. Similarly, restricting participation is more important if the aim is low effort.[12]

4 Market integration

This section studies integration of two contest markets. We assume that both countries have pre-integration markets as described above and that the only difference is the number of firms, which is n in one country and m in the other. After integration there is one market where two projects are offered to $n + m$ contestants.

We assume that after integration, the two projects are awarded simultaneously and independently. With linear costs of effort and no capacity constraints, all firms will simultaneously participate in both contests according to the results from the Tullock single prize game, but the number of contenders is now $k = n + m \geq 4$ in both contests.

4.1 Non-union benchmark

For later comparison, we briefly turn to the non-union benchmark. We have assumed that $w = 0$ in this case. Effort in the non-integrated (NI) and

[12] We have not specified any objectives for the project administrator. If the aim is simply to maximize or minimize contest effort, the results in the unionized and non-unionized cases would be the same. However, if the administrator has to balance effort levels against other considerations, the above argument might prove important.

integrated (I) cases is given by:

$$NI : S_n = \frac{n-1}{n}V, \tag{15}$$

$$I : S_k^j = \frac{k-1}{k}V, \tag{16}$$

where superscripts $j = H(ome), F(oreign)$ denotes the two projects. Firm expected profits in equilibrium are thus given by:

$$\Pi^{NI} = \frac{1}{n}V - x_i = \frac{1}{n}V - (S_n - \frac{(S_n)^2}{V}) = \frac{1}{n^2}V, \tag{17}$$

$$\Pi^{I} = \frac{2}{n+m}V - x_i^1 - x_i^2 = \frac{2}{(n+m)^2}V. \tag{18}$$

From this it is easily seen that the firms only benefit from integration if the country they reside in is sufficiently large, in terms of the number of firms, relative to the other country. Actually, the number of firms in the parent country has to be some 140% larger. Integration leads to more competition, and thus hurts firms. But the possibility of getting another project will offset this effect if there are relatively few new contenders. This will be the case when a contest market is integrated with one that is sufficiently small in comparison.

Proposition 4 *In the non-union case, market integration is never beneficial to firms of both countries. If one country is large in terms of the number of firms relative to the other country, firms residing in that country may benefit from integration.*

We do not undertake a welfare analysis in this model, as it would require a framework stipulating to what extent effort should be considered socially wasteful. Assuming that all effort is pure waste from a welfare point of view, consumers only come into consideration through some value that the authorities may assign to having the projects finished. Thus, we find that the present model is not well suited for welfare analysis.[13]

4.2 The unionized integrated economy

In the twin-prize contest, union wage setting now influences the firm's actions towards getting both projects, either of which can generate employment to

[13] Additionally, if we assume effort to be as beneficial to the authorities as it is costly to the firms, then effort by a firm is simply a transfer to the authorities. Similarly, in this set-up, lower wages constitute a transfer from unions to firms.

the union. Let P_i^j denote the probability of the $i'th$ firm getting project j given effort x_i^j. We assume, in the interest of simplicity, that the union of firm i gets utility w_i if the firm gets one project, and $2w_i$ if the firm gets both.[14]

Given these assumptions, expected utility for union i is[15]

$$E[U_i] = (P_i^H + P_i^F)w_i. \tag{19}$$

Now, the probabilities are given by $P_i^H = 1 - \frac{S_k^H}{V-w_i}$ and $P_i^F = 1 - \frac{S_k^F}{V-w_i}$ where $S_k^H = S_k^F = \frac{k-1}{\sum_{j=1}^{k}\frac{1}{V-w_j}}$. This means that the probabilities of getting the two projects are equal. Setting $P_i^H = P_i^F = P_i$, we have the $i\acute{}$th union's maximization problem given by

$$w_i = \arg\max_{w_i}[(P_i^H + P_i^F)w_i = 2P_i w_i]. \tag{20}$$

No more calculations are needed here as this maximization problem is identical to the one in the previous section, only now with $n+m$ participating firm/union pairs.[16] The optimum (symmetric) wage level is therefore given by[17]

$$w = (n+m)\frac{V}{n+m+(n+m-1)^2}. \tag{21}$$

We can now find the following:

Proposition 5 *The post integration wage level is lower than the wage in either country before integration*

Proof. Given (21), the result follows immediately from Proposition 2. ■

The result is not completely transparent. Assume that two countries of very different sizes merge to become one contest market. Then the unions

[14] This could be reasonable if the union has a large member stock operating at the competitive wage level and can draw twice the number of workers from this pool if the relevant firm is awarded two rather than one project. Insider-domination is disregarded in our set-up.

[15] The probability for the firm of getting exactly one project is $(1-P_i^H)P_i^F + (1-P_i^F)P_i^H$. Similarly, the probability of getting two projects is $P_i^H P_i^F$. Given these assumptions, $E[U_i] = [(1-P_i^H)P_i^F + (1-P_i^F)P_i^H]w_i + 2P_i^H P_i^F w_i = (P_i^H + P_i^F)w_i$

[16] A shortcoming of this set-up is that it reduces market integration to having only a 'number-of-firms' effect. However, expanding the set-up is only possible at the expense of introducing much more elaborate mathematics.

[17] Following this procedure, we can easily model each country offering more than one project. If the union gets utility hw_i if its firm manages to get h projects, wages would be the same as for one project (the elasticities determine wages). The effect of market integration on wages would then again be determined solely by the number of firms.

in the larger market face less than double the competition, but double the number of projects. At first glance, this would be a situation that could be suspected to lead unions in the large country (which constitute a majority) to demand higher wages. However, the same consideration applies here as in the case of an increased number of firms in the single prize set-up (Proposition 2). In the integrated market, every union faces a more elastic probability of success, leading them to choose lower wages. The level effect just described, however, does influence equilibrium utility, which can easily be calculated:

$$E[U_i^{NI}] = \frac{1}{n} w^A = \frac{V}{n + (n-1)^2}, \tag{22}$$

$$E[U_i^I] = \frac{2}{n+m} w^I = 2\frac{V}{(n+m-1)^2 + (n+m)}. \tag{23}$$

For large n, both in absolute and relative terms, U_i^{NI} can be approximated by $\frac{V}{n^2}$, while U_i^I similarly follows $\frac{2V}{n^2}$. Thus it is apparent that integration is beneficial to the unions of the large country. Similarly, it is not beneficial for unions of a small country.[18] For the special case of $n = m$, it can easily be demonstrated that integration is not beneficial to unions of either country. This means that under no circumstance will both unions benefit from integration.[19] We have thus illustrated:

Corollary 6 *Market integration is beneficial to unions of a country only if that country has a substantial majority of the firms that later compete in the unified market.*

This is due to the effects discussed under proposition 5. It is also worth noting that the larger country has the smaller reduction in wages after integration.

Turning to firm profitability, we have

$$\begin{aligned} E[\Pi_i^{NI}] &= P_i(V - w^A) - x_i = \frac{1}{n}(1 - \frac{n-1}{n})(V - w^A) \\ &= \frac{1}{n^2}\frac{(n-1)^2}{n + (n-1)^2} V, \end{aligned} \tag{24}$$

$$E[\Pi_i^I] = \frac{2}{(n+m)^2}\frac{(n+m-1)^2}{n+m+(n+m-1)^2} V. \tag{25}$$

[18] The validity of assuming a large number of firms in this context can be debated. However, the discussion here is intended merely as an illustration. The results pertain even for a small number of firms.

[19] To a good approximation, n has to exceed $0.5 + (2.41)m$ in order for unions of the larger country to benefit from integration. For $m = 2$, this entails $n > 5$.

Since market integration leads to lower wages, integration is more beneficial (or less detrimental) for firms in the unionized setting compared to a situation where there are no unions. Substituting $m = n$ in the above expression shows that firms of both countries can benefit if $n = m = 2$. We thus have:

Proposition 7 *With a unionized labor market and two firms in each country, firms from both countries will benefit from market integration.*

However, for more than two firms in one or both countries, the results from the non-unionized case (Proposition (4)) are valid in this context also. Thus the wage reduction caused by integration has a positive effect on firm profitability, but the effect is not large enough to increase expected profits, as equilibrium effort is rising more than in the non-union case.

5 Concluding remarks

In this paper we have used the theory of rent seeking to study unionized contest markets and the effects of market integration for contest market performance. Adding unions to a standard Tullock model of rent-seeking, we have shown that wages are decreasing in the number of firms, but for non-trivial reasons. It is furthermore demonstrated that rent seeking will be more critically dependent upon the number of contestants than would be the case without unions.

For the non-union case, we have shown that market integration would never be beneficial to firms of both countries. This is the same as the standard result for the case of oligopolistic industries. Anderson *et al.* (1989) finds that firms of at least one country would lose from opening up to free trade in such a setting. However, for the case of plant level unionized labor markets, we have shown that market integration could indeed benefit firms of both countries. The reason is that market integration leads unions to set lower wages, which contributes positively to a firm's expected profits.[20]

[20] Naylor (1998, 1999) demonstrates how this result may also be valid for oligopolistic industries. In a two-firm/two-country Cournot setup, he shows how market integration, in the form of reduced costs of trade, may benefit firms and unions alike if trade costs are already sufficiently low. However, for the transition from pre-emptive trade costs to zero trade costs (which is the relevant parallel to our discussion), firms will lose even though integration leads to lower wages.

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Appendix

Probability and wage

From section 2, we have

$$P = \frac{x_i}{S_n}, \tag{26}$$

$$x_i = S_n - \frac{S_n^2}{V - w_i}, \tag{27}$$

$$S_n = \frac{n-1}{\sum_{j=1}^{n} \frac{1}{V-w_j}}. \tag{28}$$

Thus, we can deduce:

$$\frac{\partial S_n}{\partial w_i} = -S_n^2 \frac{1}{n-1}(-1)(\frac{1}{V-w_i})^2(-1) = -S_n^2 \frac{1}{(n-1)(V-w_i)^2}. \tag{29}$$

Furthermore,

$$\begin{aligned} \frac{\partial x_i}{\partial w_i} &= \frac{\partial S_n}{\partial w_i} - \frac{(V-w_i)2S_n \frac{\partial S_n}{\partial w_i} + S_n^2}{(V-w_i)^2} \\ &= \frac{\partial S_n}{\partial w_i}(1 - \frac{2S_n}{V-w_i}) - \frac{S_n^2}{(V-w_i)^2}. \end{aligned} \tag{30}$$

This leaves

$$\begin{aligned} \frac{\partial P}{\partial w_i} &= -\frac{x_i}{S_n^2}\frac{\partial S_n}{\partial w_i} + \frac{1}{S_n}\frac{\partial x_i}{\partial w_i} \\ &= -[-\frac{x_i}{S_n^2} + \frac{1 - \frac{2S_n}{V-w_i}}{S_n}]S_n^2 \frac{1}{(n-1)(V-w_i)^2} - \frac{1}{S_n}\frac{S_n^2}{(V-w_i)^2} \\ &= [x_i - S_n(1 - \frac{2S_n}{V-w_i})]\frac{1}{(n-1)(V-w_i)^2} - \frac{S_n}{(V-w_i)^2} \\ &= [(S_n - \frac{S_n^2}{V-w_i}) - S_n(1 - \frac{2S_n}{V-w_i})]\frac{1}{(n-1)(V-w_i)^2} - \frac{S_n}{(V-w_i)^2} \\ &= \frac{S_n^2}{V-w_i}\frac{1}{(n-1)(V-w_i)^2} - \frac{S_n}{(V-w_i)^2} \\ &= \frac{S_n}{(V-w_i)^2}[\frac{S_n}{(n-1)(V-w_i)} - 1]. \end{aligned} \tag{31}$$

In relation to Proposition 2, we substitute the value for S_n and obtain

$$\frac{\partial P}{\partial w_i} = (n-1)\frac{\frac{1}{\sum_{j=1}^{n}\frac{1}{V-w_j}}}{(V-w_i)^2}[\frac{\frac{1}{\sum_{j=1}^{n}\frac{1}{V-w_j}}}{(V-w_i)} - 1]. \tag{32}$$

This expression provides us with a rationale behind the result that more firm/union pairs induce lower wages. As we see, the probability's dependence on wages is directly dependent upon the number of firms. However, to complete this line of thought, one has to account for the alterations in $\sum_{j=1}^{n}\frac{1}{V-w_j}$. This is not included.